NDSU Math 166 Fall 2008 **Name**_____

Final Exam (Martin): Tuesday December 16, 3:15-5:15 pm **Instructor**_____

You may use a graphing calculator on any problem.
Read each question carefully before you do any work. Be sure to answer what the question asks.
Use complete sentences to explain things. You must show valid work or reasoning to receive full credit for answers. All work and answers should be written in a large, blue examination booklet. Each of the problems is worth 10 points.

1. "True" and "False" questions. Support your responses:
   (a) If $\lim_{n\to\infty} a_n \neq 0$, then the sequence $\{a_n\}_{n=1}^{\infty}$ is divergent.
   (b) $\int_{-1}^{1} \frac{dx}{x^2} = -2$.
   (c) The series $1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$ is the power series, the Taylor series, and the Maclaurin series for the function e^x at 0.
   (d) If a_n is convergent to 0, then the series $\sum_{n=1}^{\infty} a_n$ must also be convergent.
   (e) $\frac{x^2+4}{x^2(x-4)}$ can be put in the form $\frac{A}{x^2} + \frac{B}{x-4}$.

2. Show how to evaluate any three of these integrals without tables or calculators:
   (a) $\int \sin^2 x\, dx$ (b) $\int \sin^3 x\, dx$ (c) $\int e^x x^2\, dx$ (d) $\int \frac{x+5}{x^2+x-2}\, dx$

3. Find the volume of the solid formed by rotating the curve $y = \sin x$, $0 \le x \le \pi$, about the *y*-axis.

4. Use the Integral Test to determine whether or not the series $\sum_{n=1}^{\infty} ne^{-n^2}$ converges. Show all of your work.

5. Write integrals to find
   (a) the arc length of and
   (b) the area enclosed by
the cardioid $r = 1 + \sin\theta$. Show your work, but **DO NOT EVALUATE EITHER INTEGRAL.**

Problems 6-10 are printed on the back of this page

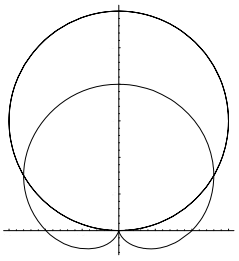
6. Find the slope of the tangent to the curve $x = t - \sin t,\ y = 1 - \cos t$ at the point where $t = \pi/6$.

7. Approximate $\ln(5)$ using $\ln(x) = \int_1^x \frac{dt}{t}$ and Simpson's Rule with $n = 4$. Simpson's rule is:

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right],$$

$$n \text{ even and } \Delta x = \frac{b-a}{n}$$

8. Set up an integral that gives the area inside of the circle $r = 3\sin\theta$ and outside of the cardioid $r = 1 + \sin\theta$ (see figure). ***You do not need to evaluate the integral.***



9. Test each series for convergence. If the series is convergent, state whether each convergent series is *absolutely* or *conditionally* convergent.

   (a) $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n \ln n}$ (b) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{e^n}$

10. (a) Show that $\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$. You may use any valid method, but must show your reasoning.

    (b) Find the *radius* and *interval* of convergence for the series in part (a).